Wayne Lafreniere
Differentiated Lesson for 3/25

Topic: Utilizing the graphing calculator to better understand quadratic equations.

Differentiation Strategy: Tiered Lesson

Lesson Flow: Students will use graphing calculators to explore how changes in the coefficients a, b, and the constant, c in quadratic equations ($ax^2 + bx + c$) affect the graph. Once basic understanding is attained, students will be challenged to figure out how to copy a pattern of graphs or create their own design.

Do now: Graph and sketch the basic quadratic equation $Y = x^2$.

Once students have demonstrated the ability to enter and graph equations, they will be given one of three tiered tasks, providing each with the appropriate support to guide them through the lesson. The ultimate goal is that of understanding what in the equation "moves" the graph in a prescribed manner. This will hopefully enable students to recognize the type of equation that has generated a specific graph.

Tiers

One: Students who take longer to pick up certain concepts will be provided with support materials that will "lead" them through the lesson, with support from the teacher. Students will enter equations into the graphing calculator and match that graph to a sketch, already made for them. The teacher will ask probing questions to develop understanding from their observations. Once they have gained enough understanding, they will be challenged to create a design out of graphs by generating appropriate equations.

Two: Students on level will follow the lesson outlined in the text book. The lesson is designed to develop concepts through questions making students draw conclusions from observations of their work, with a minimal amount of support from the teacher. Once they have completed the lesson, they should have enough understanding to create a design out of graphs by generating appropriate equations.

Three: Students who are above level will be challenged to duplicate existing designs in an inquiry study that encourages trial and error. If need be, they will be directed to parts of the basic text lesson to clarify areas where they demonstrate a lack of complete understanding.

Assessment: All students will be required to generate a "design" of their own choice along with an explanation of how they generated the design and how they knew what, in the equations, would yield the desired pattern. Students will choose a pattern with which they are confident in their own understanding. Projects will be developed as "bulletin board projects."

Focus: Were students fully engaged and capable of successfully completing the task within the framework provided?

Name_____ Class_____

Match the following equations to the appropriate graph.
Determine the value of a, b, and c. (If a term does not appear, the coefficient is 0.)

Group I

$y = x^2 + 3$	$y = 4x^2$	$y = x^2 + 1$	$y = -1/2x^2$
$y = x^2 - 1$	$y = -4x^2$	$y = x^2 - 3$	$y = 1/2x^2$

Group II

$y = x^2 + 2$	$y = -2x + 2$	$y = x^2 - 3x + 2$	$y = x^2 - 6x + 2$
$y = -2x^2 - 3x + 2$	$y = -2x^2 - 6x + 2$	$y = x^2 + 3x + 2$	$y = x^2 + 6x + 2$
$y = -2x + 3x + 2$	$y = -2x^2 + 6x + 2$		

When "a" is positive, the parabola opens_____.

When "a" is negative, the parabola opens_____.

When the |a| increases, the width of the parabola _____.

When the |a| decreases, the width of the parabola _____.

In group I, the value of "b" is ____.

As long as the value of "b" is _____, (fill in the answer from the question above)

when "c" is 0, (see your very first graph), the vertex is at the _____.

when "c" is positive, the vertex moves _____ on the _____ axis.

when "c" is negative, the vertex moves _____ on the _____ axis.

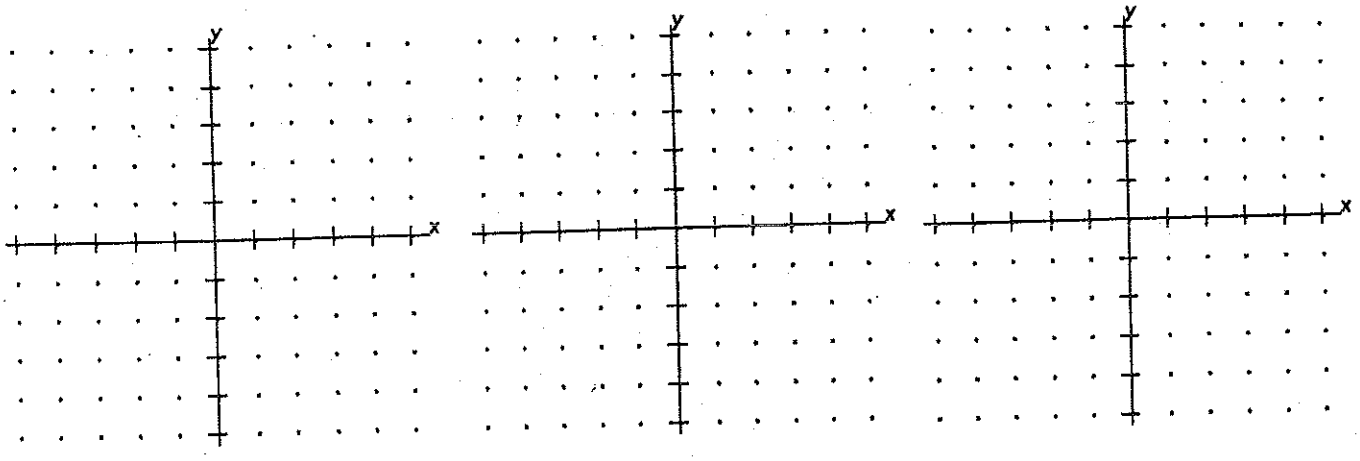
When "a" is positive and "b" is positive, increasing the |b| moves the vertex _____ and to the _____.

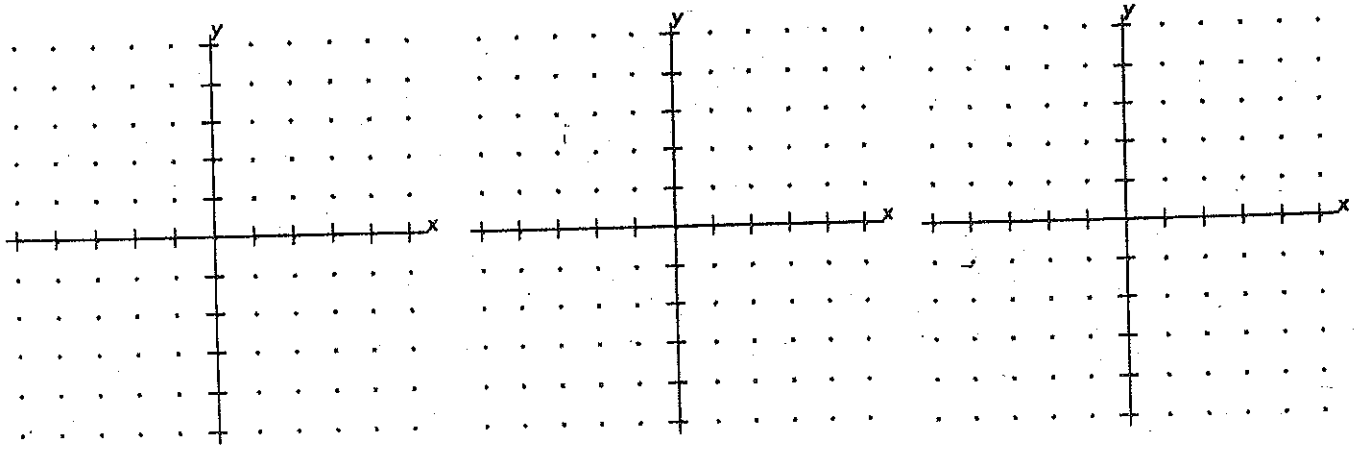
When "a" is positive and "b" is negative, increasing the |b| moves the vertex _____ and to the _____.

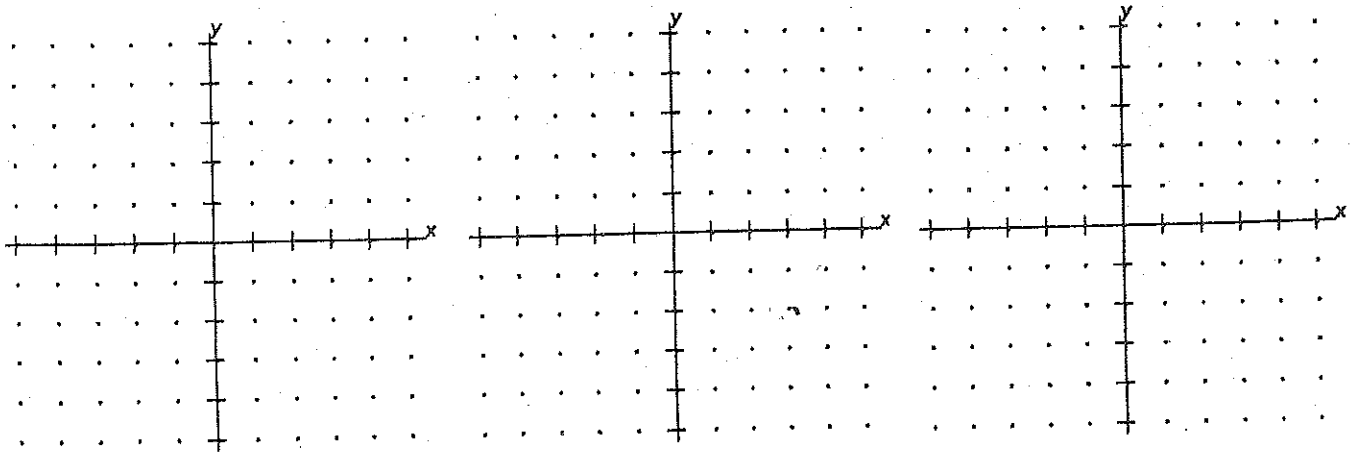
When "a" is negative and "b" is positive, increasing the |b| moves the vertex _____ and to the _____.

When "a" is negative and "b" is negative, increasing the |b| moves the vertex _____ and to the _____.

Name_____ Class_____







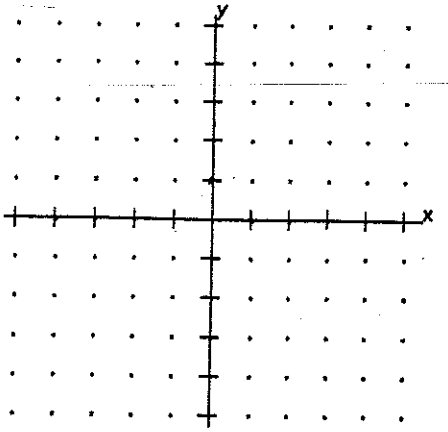
Name_____ Class_____

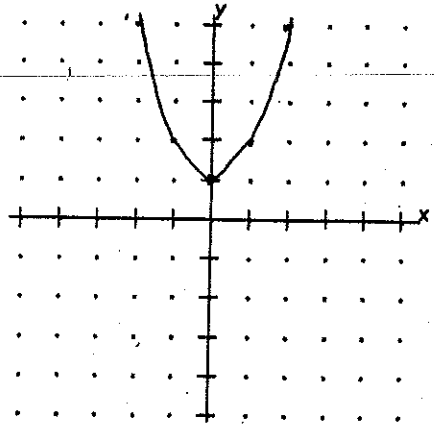
Group I

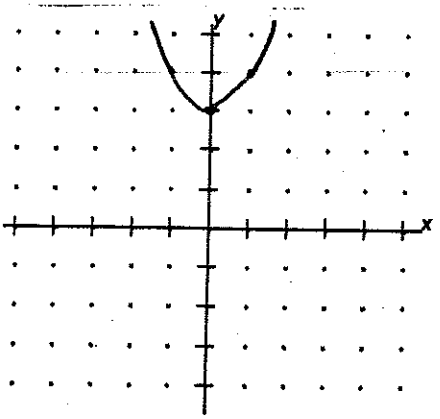
Equation $y = x^2$
a = 1 b = 0 c = 0

Equation_____
a = b = c =

Equation_____
a = b = c =



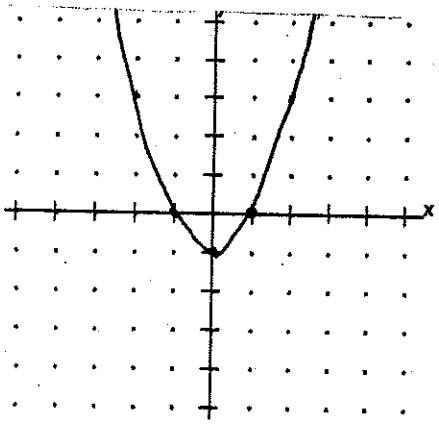


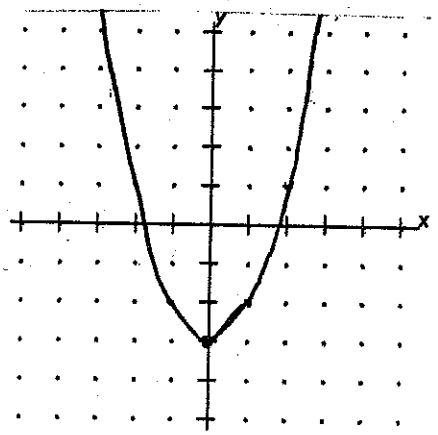


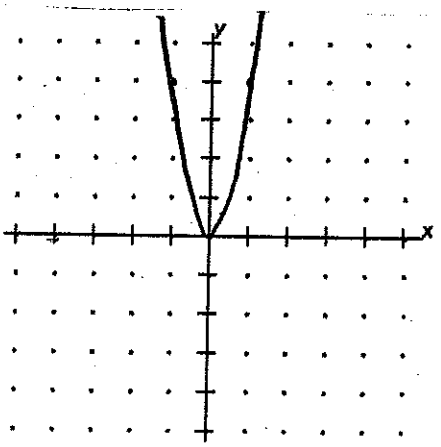
Equation_____
a = b = c =

Equation_____
a = b = c =

Equation_____
a = b = c =



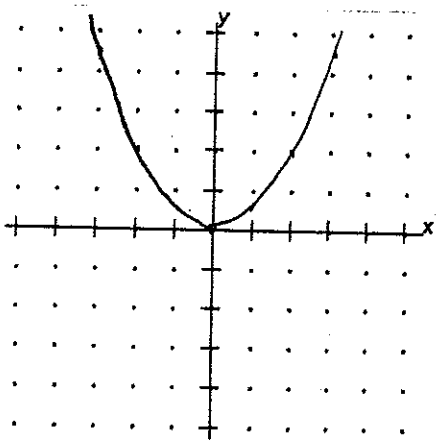


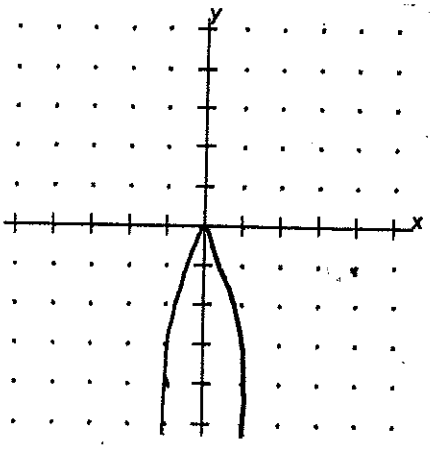


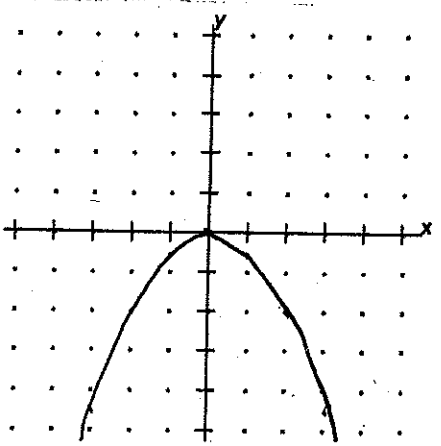
Equation_____
a = b = c =

Equation_____
a = b = c =

Equation_____
a = b = c =



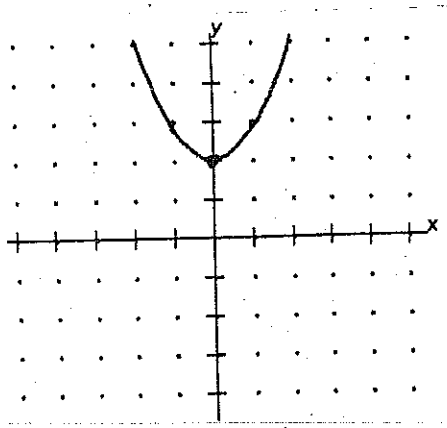




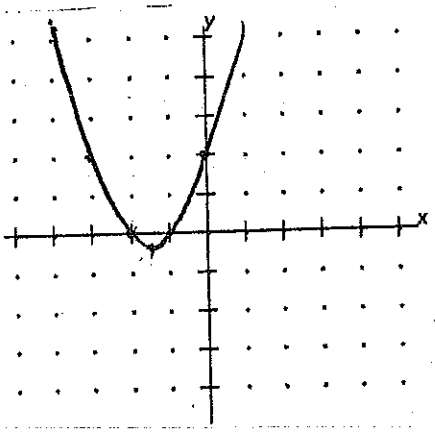
Name_____ Class_____

Group II

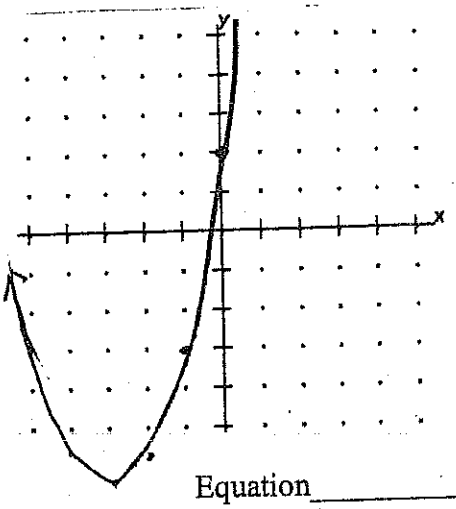
Equation_____
a = b = c =



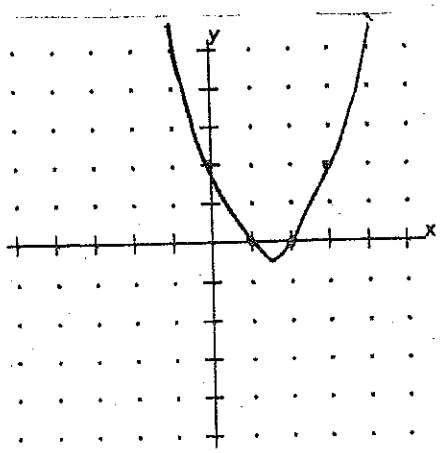
Equation_____
a = b = c =



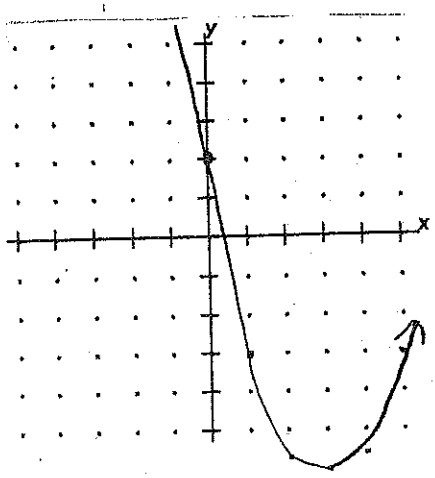
Equation_____
a = b = c =



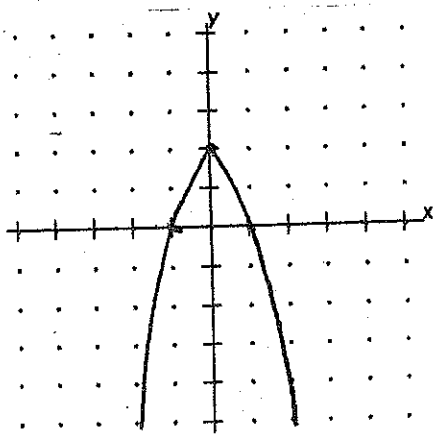
Equation_____
a = b = c =



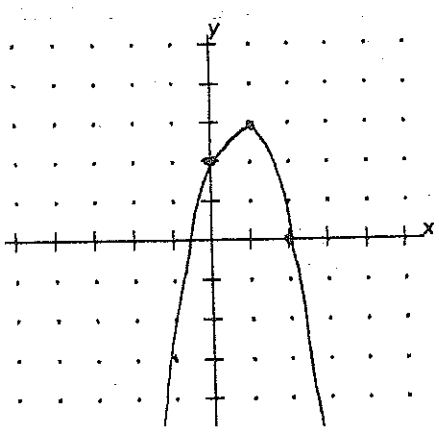
Equation_____
a = b = c =



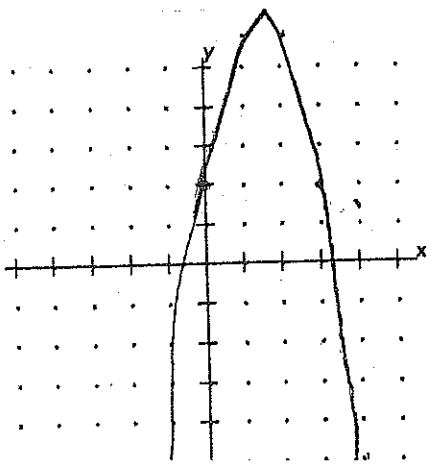
Equation_____
a = b = c =



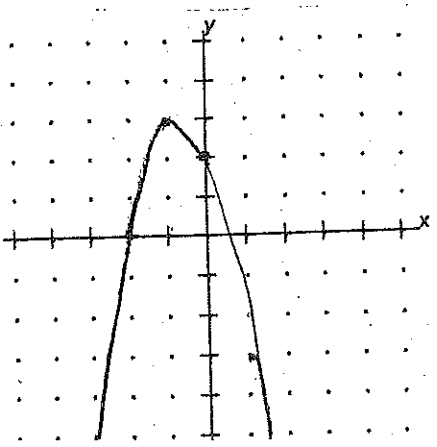
Equation_____
a = b = c =



Equation_____
a = b = c =



Equation_____
a = b = c =



Develop & Understand: A

1. Complete Parts a–c for each group of quadratic equations.

Group I	Group II	Group III	Group IV
$y = x^2$	$y = x^2$	$y = x^2$	$y = -x^2$
$y = x^2 + 1$	$y = x^2 - 1$	$y = 2x^2$	$y = -2x^2$
$y = x^2 + 3$	$y = x^2 - 3$	$y = \frac{1}{2}x^2$	$y = -\frac{1}{2}x^2$

a. Graph the three equations in the same window of your calculator. Choose a window that shows all three graphs clearly. Make a sketch of the graphs. Label the minimum and maximum values on each axis. Also, label each graph with its equation.

b. For each group of equations, write a sentence or two about how the graphs are similar and how they are different.

c. For each group of equations, give one more quadratic equation that also belongs in that group.

2. Describe how the graphs in Group I are like the graphs in Group II and how they are different.

3. Describe how the graphs in Group III are like the graphs in Group IV and how they are different.

4. Use what you learned in Exercises 1–3 to predict what the graph of each equation below will look like. Make a quick sketch of the graphs on the same set of axes. Be sure to label the axes. Also, label each graph with its equation. Check your predictions with your calculator.

a. $y = x^2 + 2$ **b.** $y = 3x^2 + 2$

c. $y = -3x^2 + 2$ **d.** $y = \frac{1}{2}x^2 - 3$

5. All of the equations you have seen in this exercise set are in the form $y = ax^2 + bx + c$, but the coefficient b is equal to 0. Explain how the values of a and c affect the graph of an equation.

In Exercises 1–5, you probably saw that equations of the form $y = ax^2 + c$ have their highest or lowest point at the point $(0, c)$. The highest or lowest point of a parabola is called its **vertex**.

Not all parabolas have their vertices on the y-axis. In the following exercises, you will look at the properties of an equation that determine where the vertex of its graph will be.

Develop & Understand: B

Consider these four groups of quadratic equations.

Group I	Group II	Group III	Group IV
$y = x^2 + 2$	$y = x^2 + 2$	$y = -2x^2 + 2$	$y = -2x^2 + 2$
$y = x^2 + 3x + 2$	$y = x^2 - 3x + 2$	$y = -2x^2 + 3x + 2$	$y = -2x^2 - 3x + 2$
$y = x^2 + 6x + 2$	$y = x^2 - 6x + 2$	$y = -2x^2 + 6x + 2$	$y = -2x^2 - 6x + 2$

6. In each group, all of the equations have the same values of a and c. Use what you learned about the effects of a and c to make predictions about how the graphs in each group will be alike.

7. For each group of equations, complete Parts a–c.

a. Graph the three equations in the same window of your calculator. Choose a window that shows all three graphs clearly. Make a sketch of the graphs. Remember to label the axes, and label the graphs with their equations.

b. Were your predictions in Exercise 6 correct?

c. For each group of equations, write a sentence or two about how the graphs are similar and how they are different.

8. What patterns can you see in how the locations of the parabolas change as b increases or decreases from 0?

Share & Summarize

1. Imagine moving the graph of $y = -2x^2 + 3$ up 2 units without changing its shape. What would be the equation of the new parabola?

2. Briefly describe or make a rough sketch of the graph of $y = -\frac{1}{2}x^2 - 2$.

3. For each quadratic equation, tell whether the vertex is on the y-axis. Explain how you know.

a. $y = \frac{1}{2}x^2 - 3$

b. $y = x^2 - 3x + 1$

c. $y = -x^2$

d. $y = -3x^2 + x + 13$

e. $y = -x^2 + 4$

f. $y = 7x^2 + 3x$

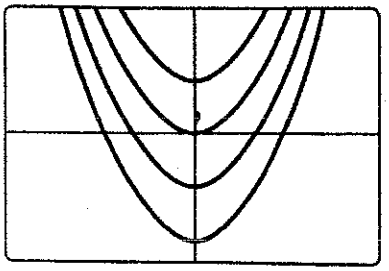
Investigation 5 Graph Design Puzzles

Materials

- graphing calculator

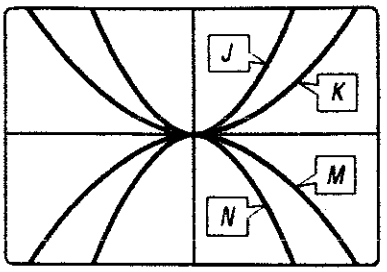
These simple designs are made from the graphs of quadratic equations.

Design A



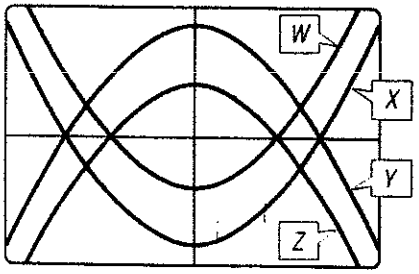
In this design, the parabolas are equally spaced and have their vertices on the y-axis.

Design B



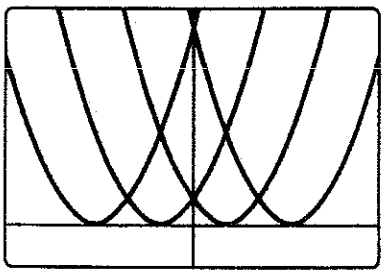
Here each parabola has a vertex at the origin. Parabolas J and N are the same width. Parabolas K and M have the same width.

Design C



In this design, Parabolas W and Z are the same width, and their vertices are the same distance from the origin. Parabolas X and Y are the same width, and their vertices are the same distance from the origin.

Design D



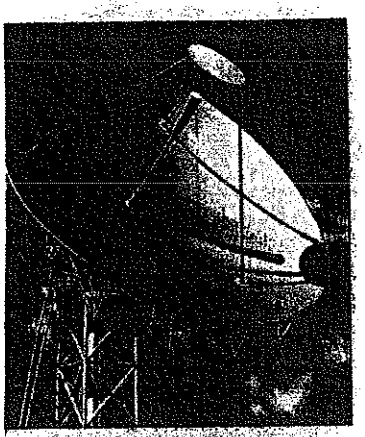
In this design, the vertices of the four parabolas are equally spaced along the x-axis. Two of those points are to the left of the origin, and two are to the right.

Try It Out

With your group, choose Design A, B, or C. Try to create the design on your calculator. Use equations in the form $y = ax^2 + bx + c$, and experiment with different values of a, b, and c. You may need to adjust the viewing window to make the design look the way you want.

1. When you have created the design, make a sketch of the graph. Label each curve with its equation. Also label the axes, including the maximum and minimum values on each axis.

2. Different sets of equations and window settings can give the same design. Compare your results for Question 1 with the other members of your group or with other groups who chose your design. Did you record the same equations and window settings?



Real-World Link
The design of satellite dishes is based on the parabola.

Try It Again

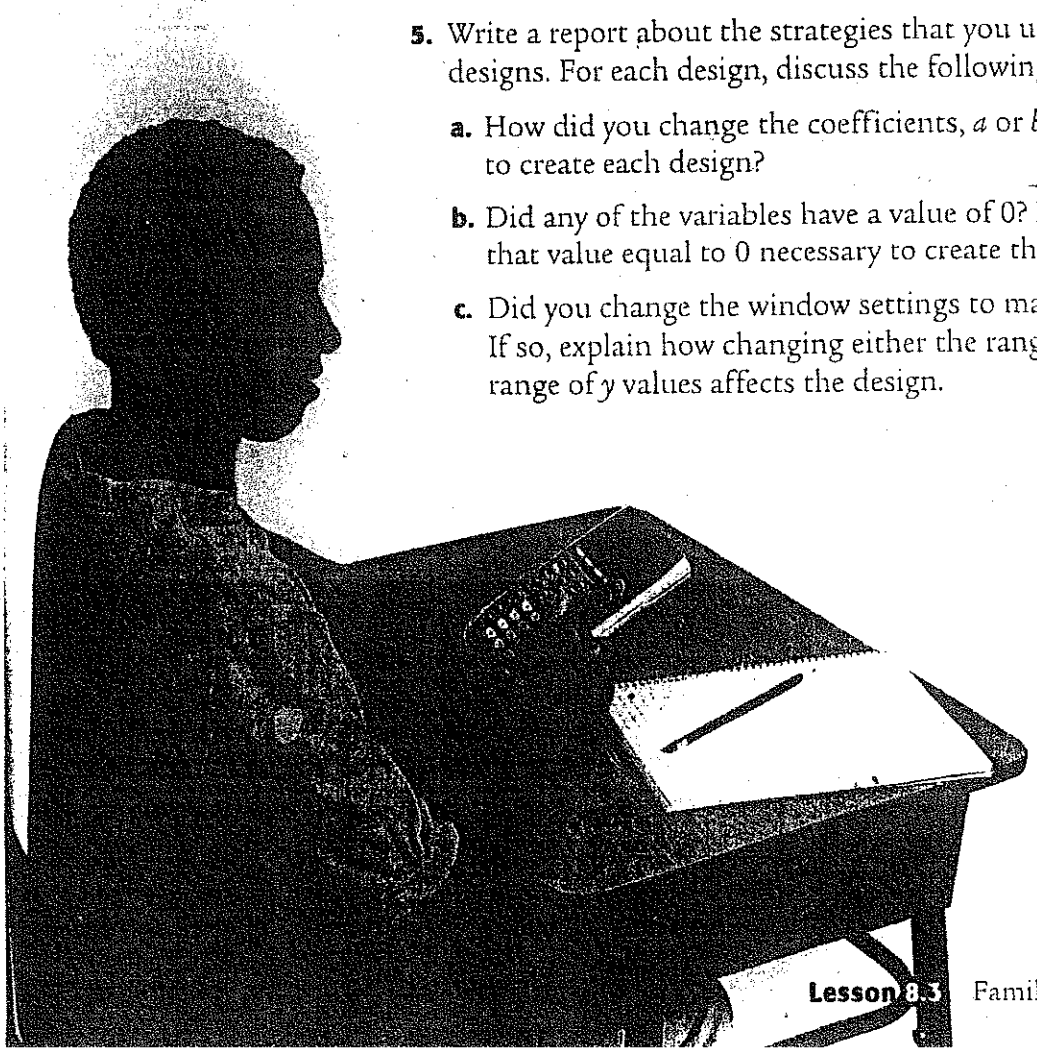
3. Now create each of the other three designs. For each design, make a sketch. Record the equations and window settings that you used. You may find Design D to be a challenge.

Take It Further

4. Work with your group to create a new design from the graphs of four quadratic equations.
   a. Make a sketch of your design. *On a separate sheet of paper,* record the equations and window settings that you used.
   b. Exchange designs with another group. Try to re-create the design.

What Did You Learn?

5. Write a report about the strategies that you used to re-create the designs. For each design, discuss the following points.
   a. How did you change the coefficients, a or b, or the constant, c, to create each design?
   b. Did any of the variables have a value of 0? If so, why was having that value equal to 0 necessary to create the design?
   c. Did you change the window settings to make any of the designs? If so, explain how changing either the range of x values or the range of y values affects the design.



Quadratic Patterns

Task: In a coordinate plane:
Create a design, using *quadratic* equations.
List each equation used to generate the design.
Differentiate each *parabola* and its'corresponding equation with a separate color.
On a separate page, **discuss** how you created your design with specific detail about what you did to the various coefficients of each term or to the constant "c" to achieve the design. Be specific in describing how changing the coefficients "moved" the graph.

Projects will be graded based on the accompanying rubric.

Rubric:

4) Work is extremely neat and well organized.
There is a recognizable pattern to the design.
Quadratic equations are used.
Graphs accurately match the equations.
Graphs and equations are color coded.
The discussion is specific, clear and complete.

3) Work is neat and organized.
There is a recognizable pattern to the design.
Quadratic equations are used.
There may be a slight error in the graphs.
Graphs and equations are color coded.
The discussion is specific, relatively clear, but may be missing some information.

2) Work lacks neatness or is not well organized.
The pattern in the design may not be perfect.
Quadratic equations are used.
There may be multiple errors in the graphs.
Graphs and equations are not color coded.
The discussion is not specific, lacks clarity and/or completeness.

1) Work is sloppy and disorganized.
There is no discernable pattern.
Not all equations are quadratic.
There are many errors in the graphs.
Graphs and equations are not color coded.
The discussion is confusing or completely missing.