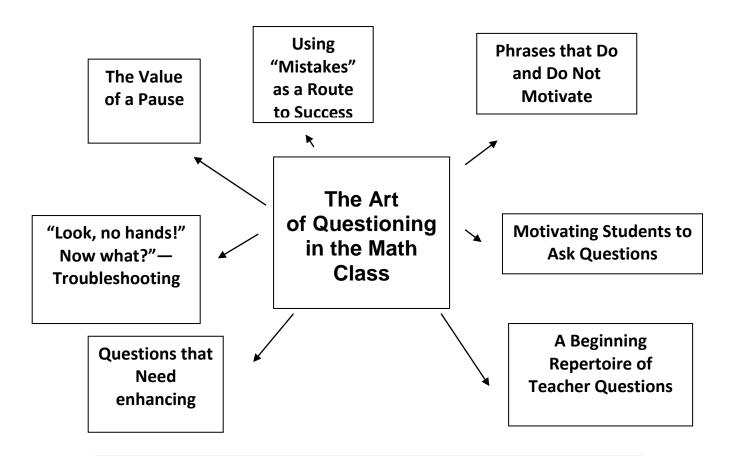
PROMOTING MATHEMATICAL THINKING AND DISCUSSION WITH EFFECTIVE QUESTIONING STRATEGIES



Recall: Five Productive Talk Moves

- 1. Revoicing
- 2. Asking Students to Restate Someone Else's Reasoning
- Asking Students to Apply Their Own Reasoning to Someone Else's Reasoning
- 4. Prompting Students for Further Participation
- 5. Using Wait Time

Some Productive Talk Formats

- 1. Whole-Class Discussion
- 2. Small-Group Discussion
- 3. Partner Talk

What Do We Talk About?

- 1. Mathematical Concepts
- 2. Computational Procedures
- 3. Solution Methods and Problem-Solving Strategies
- 4. Mathematical Reasoning
- 5. Mathematical Terminology, Symbols, and Definitions
- 6. Forms of Representation

What Teachers Should Know About Questioning in the Math Class

Asking questions that motivate student reflective thinking is an art. If our lessons are to be effective, we need to develop this art. It takes practice. As with most arts, there is not a set of hard and fast rules that work in all situations all of the time, but here are some general effective ideas to keep in mind for most times.

1. A "Try-to" List:

- Try to use effective pauses and wait time.
- Try to avoid frequent questions which require only a yes/no answer or simple recall.
- Try to avoid answering your own questions.
- Try to follow up student responses with questions and phrases such as, "why?" or "tell me how you know" or "think about how you can put Jim's response into your own words."
- Try to avoid directing a question to a student mainly for disciplinary reasons.
- Try to follow up a student's response by fielding it to the class or to another student for a reaction.
- Try to avoid giveaway facial expressions to student responses.
- Try to make it easy for students to ask a question at any time.
- Try to ask the question before calling on a student to respond.
- Try not to call on a particular student immediately after asking a question.
- Try to ask questions that are open-ended.
- Try not to label the degree of difficulty of a question.
- Try to leave an occasional question unanswered at the end of the period.
- Try to replace or enhance "lectures" with a set of appropriate questions.
- Try to keep the students actively involved in the learning process.
- 2. **Questions to seldom ask**: [The point here is that even though you *do* want to know the answers to these questions, the way these questions are phrased probably won't get you very far in learning what you want to know.]
 - "How many of you understood that?"
 - "Everybody see that?"
 - "You want me to go over that again?"
 - "This is a right triangle, isn't it?"
 - "Do you have any questions?"

- 3. **Phrases That Encourage Participation:** It's useful to have a handful of effective ways to start your questions that will motivate all students to participate. Here are some to try. What others can you think of?
 - "Don't raise your hand--yet; just think about a possible answer. I will give you a minute . . . "
 - "Everyone—picture this figure in your mind. Is it possible to sketch a possible counterexample to this statement? . . . I will walk around to look at your work and select 3 students to share their results with the class."
 - "Find an example for this statement and write it down. In just a minute I will tell you possible ways to check your example to see if it indeed makes the statement true."
 - "Put the next step on your paper and write a reason to justify this step. Raise your hand when you are ready and I will be around to check in on you."
- 4. **Phrases That May Fail to Motivate:** There are some questions that you might want to avoid. Why? Because often you end up answering your own questions . . . and "permitting" students NOT to participate—that is, students are not required to take responsibility to develop a response depending how the question is phrased.
 - "Does someone know if . . . "
 - "Can anyone here give me an example of . . . "
 - "Who knows the difference between . . . "
 - "Someone tell me the definition of . . . "
 - "OK, who wants to tell me about . . . "
- 5. **Questions That Need Enhancing:** Some common types of questions need some special care if they are to be useful in the math classroom. Otherwise, these questions do not provide much information to check students' reasoning.
 - Yes-No questions
 - True-false questions
 - One-word-answer questions
 - Questions that fail to motivate

Phrases for enhancing questions:

- "Tell me more about what you were thinking."
- "How did you decide that?"
- "Elaborate for others in the class so they can check their thinking."
- "Can you justify that?"
- "Give us your insights about arriving at the answer."
- "What steps did you take?"
- "Tell us more about what you're thinking."
- "What made you think of that?"
- "To a person on the street who doesn't speak "Math," tell how you decided that . . . "
- 6. **The Value of a Pause:** An extremely valuable skill within the art of questioning is in knowing how to build in a pause at the appropriate time. This means both students and teacher need to value a pause.

Note: Pausing is not always comfortable. However, in a classroom where both the teacher and the students pause at appropriate times, research has shown that some exciting things can happen:

- The length of student responses increase
- students' confidence increases
- weaker students contribute more
- there's a greater variety of student responses
- discipline problems decrease
- creative responses increase

Some suggestions about when to pause,	, who pauses,	and the reas	soning behind	the
pause:				

- •
- •
- •

7. Encouraging Your Students to Ask Questions

Instead of asking questions this way:

- "Are there any questions?"
- "Do you have any questions?"
- "You don't have any questions, do you?"
- "Would anyone like to see that again?"

You might try asking questions this way:

- "Okay--What questions do you have?"
- "Now, ask me some questions."
- "Now, what questions may I answer?"
- "Give me your questions!"

Note the subtle difference. The first set sounds as if you don't want questions; the second set implies that you both want and expect questions.

Note: Now, when students do ask questions, instead of always providing quick answers, you can help to keep the class student-centered by considering the following:

- Students should practice directing questions to other students-not only to the teacher.
- The teacher should pause to permit other students to develop an answer to the question not immediately jump in and answer the question.
- The teacher should remind students that the question is for all members of the class.
- To ensure that they focus on it, students might write down the question in their notes. They could then write a possible solution or response to the question.

8. Troubleshooting: "Look, no hands! Now what?"

First, analyze the situation: When there are no hands, it may be the case that no one has a question and that everyone understands. But it also may be the case that:

- No one knows the answer or has a response.
- The level of concern is low. (No one cares.)
- The question is not understandable.
- There was insufficient time for students to formulate an acceptable response.
- The question is too difficult at this stage of the conceptual development.
- No one is confident enough to give a response.
- Students are afraid of being wrong, and of what the teacher might say or do if they give an incorrect response.

Second: Depending on how you analyze the situation, there are several possible things to consider when addressing the "no hands" scenario.

- What could you do, for instance, if the level of concern for answering the question is low?
- What might you try if you suspect that students do not understand but are hesitant to ask a question?
- Keep in mind that students need to feel safe to express that they do not understand.

9. More Troubleshooting

Analyzing the Situation	Some Ideas for Troubleshooting
1. My Students Won't Talk	
2. The Same Few Students Do All the Talking	
3. Should I Call on Students Who Do Not Raise Their Hands?	
4. My Students Will Talk, But They Won't Listen	
5. "Huh?"—Responding To Incomprehensible Contributions	

Analyzing the Situation	Some Ideas for Troubleshooting
6. Brilliant, But Did Anyone Understand?	
7. I Have Students at Different Levels	
8. What Should I Do When They're Wrong?	
 This Discussion Is Not Going Anywhere—Or At Least Not Where I Planned 	
10. Answers/Responses Are Superficial	
11. What If The First Speaker Gives The Right Answer?	
12. What Can I Do For English- Language Learners?	

Beginning Repertoire of Teacher Questions

1. Initial eliciting of students' thinking—

	•	Does anyone have a solution they would like to share?
	•	Please raise your hand when you are ready to share your solution.
	•	What did you come up with? What are you thinking?
	•	Be ready to explain the solution you got.
	•	Please explain to the rest of the class how you got your answer,
	•	How did you begin working on this problem?
	•	What have you found so far?
	•	Would anyone be willing to explain their solution?
	•	Can you point to a part of this problem that was difficult?
	•	What are some ideas you had?
	•	Raise your hand if you have a different idea.
	•	Did anyone approach the problem in a different way?
	•	What do you already know about?
2. F	Probi	ng students' answers to—
		Figure out what a student means or is thinking when you don't understand what
		they are saying
	>	,
		Check whether right answers are supported by correct understanding
		,
		Check whether right answers are supported by correct understanding
	>	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking
	•	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do?
	•	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do? How do you know?
	•	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do? How do you know? Why did you?
	•	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do? How do you know? Why did you? How did you get? Could you use [materials] to show how that works?
	•	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do? How do you know? Why did you? How did you get? Could you use [materials] to show how that works? What led you to that idea?
	•	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do? How do you know? Why did you? How did you get? Could you use [materials] to show how that works?
	•	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do? How do you know? Why did you? How did you get? Could you use [materials] to show how that works? What led you to that idea? Walk us through your steps. Where did you begin?
	•	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do? How do you know? Why did you? How did you get? Could you use [materials] to show how that works? What led you to that idea? Walk us through your steps. Where did you begin? Please give an example.
	•	Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do? How do you know? Why did you? How did you get? Could you use [materials] to show how that works? What led you to that idea? Walk us through your steps. Where did you begin? Please give an example. Would you please repeat what you said about that?
		Check whether right answers are supported by correct understanding Probe wrong answers to understand student thinking Explain what you have done so far? What else is there to do? How do you know? Why did you? How did you get? Could you use [materials] to show how that works? What led you to that idea? Walk us through your steps. Where did you begin? Please give an example. Would you please repeat what you said about that? Say a little more about your idea.

• Could you explain a little more about what you are thinking?

Can you explain that in a different way?What do you notice when _____?

3. To help when students get stuck—

- How would you describe the problem in your own words?
- What do you know that is not stated in the problem?
- What facts do you have?
- Could you try it with simpler numbers? Fewer numbers? Using a number line?
- What about putting things in order?
- Would it help to create a diagram? Make a table? Draw a picture?
- Can you guess and check?
- What did other members of your group try?
- What do you already know that could help you figure that out?

4. Focusing students to listen and respond to others' ideas—

	 What do other people think?
	 What do other people think about what said? Do you agree or disagree with the idea?
	 Would someone be willing to add on to what said?
	What do you think means by that?
	 How does what said go along with what you were thinking?
	How could you explain what said in a different way?
	Can you repeat what just said in your own words?
	Why do you think did it that way?
	Why is it okay for to do that?
	Who can explain this using's idea?
	 Does anyone have the same answer but a different way to explain it?
	Can you convince us that your answer makes sense?
	 Can anybody see what method might have used to come up with that solution?
	 How do you think got his/her solution?
5.	. Supporting students to make connections (e.g., between a model and a mathematical
	idea or a specific notation)—
	 How is 's method similar to (or different from) 's method?
	 How does [one representation] correspond to [another representation]?
	 How does [one representation] correspond to [another representation]? Can you think of another problem that is similar to this one?
	 How does [one representation] correspond to [another representation]? Can you think of another problem that is similar to this one? How does that match what you wrote on the board?
	 How does [one representation] correspond to [another representation]? Can you think of another problem that is similar to this one? How does that match what you wrote on the board? Can you explain your representation?
	 How does [one representation] correspond to [another representation]? Can you think of another problem that is similar to this one? How does that match what you wrote on the board? Can you explain your representation? Can you use the [representation] to explain what you are thinking?
	 How does [one representation] correspond to [another representation]? Can you think of another problem that is similar to this one? How does that match what you wrote on the board? Can you explain your representation? Can you use the [representation] to explain what you are thinking? How is this similar to what we learned about?
	 How does [one representation] correspond to [another representation]? Can you think of another problem that is similar to this one? How does that match what you wrote on the board? Can you explain your representation? Can you use the [representation] to explain what you are thinking?

	bes that relate to what said!
How ca	an we make a [picture, graph, model, chart] of this solution?
What p	part of the problem/solution does this [pointing to a particular part of representation]
repres	
• '	
To guido stu	idents and ancourage mathematical reflection and reasoning (e.g., make
_	idents and encourage mathematical reflection and reasoning (e.g., make
conjectures	, state definitions, generalize, prove)
 Can yo 	u explain the method you used?
 Does t 	his method always work?
	oes that work in this case?
•	do you think that would be true?
	notice any patterns?
•	do these solutions have in common?
	is method be used for other problems?
	do we mean when we say in math class?
	math terms help us to talk about that? Did you learn any new words today?
	do you mean by? Can you give a definition?
Does t	his match our reasoning? How?
 Have v 	ve found all the possible answers?
 How d 	o you know it works in all cases?
What a	about [counterexample]?
How w	ould you describe's method?
	u represent the solution in another way?
•	this problem as an example, what can you say about problems like this in general?
_	are the main ideas that you learned about today?
• Wilde	The the main faces that you rearried about today.
Extending	hudouts' surrout thinking and accessing how for thou can be stratched
extending s	tudents' current thinking, and assessing how far they can be stretched—
Can yo	u think of another way to solve this problem?
 Can yo 	u use this same method to solve?
What \	would happen if the numbers were changed to?
What i	f the problem was like this instead: [give slight variation of problem]?
	eone said [wrong answer], how would you respond?
	otice/know then what does that mean for?
	u predict the next one?
•	·
• Can yo	u think of another problem that could be solved with this method?

6.

7.